Model Fitting using Excel and Gnuplot

Biochemistry Boot Camp 2021
Session #4
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Think and Discuss

What is a scientific model?

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Properties of Models

- Explain an <u>observable quantity</u> (e.g. measured heat, growth rate, etc.)

- Express quantity in terms of understandable <u>parameters</u> and fundamental constants (equilibrium constant, rate constants, etc.)

- Should be <u>predictive</u> (so we can test the model)

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Examples: Biochemical Models
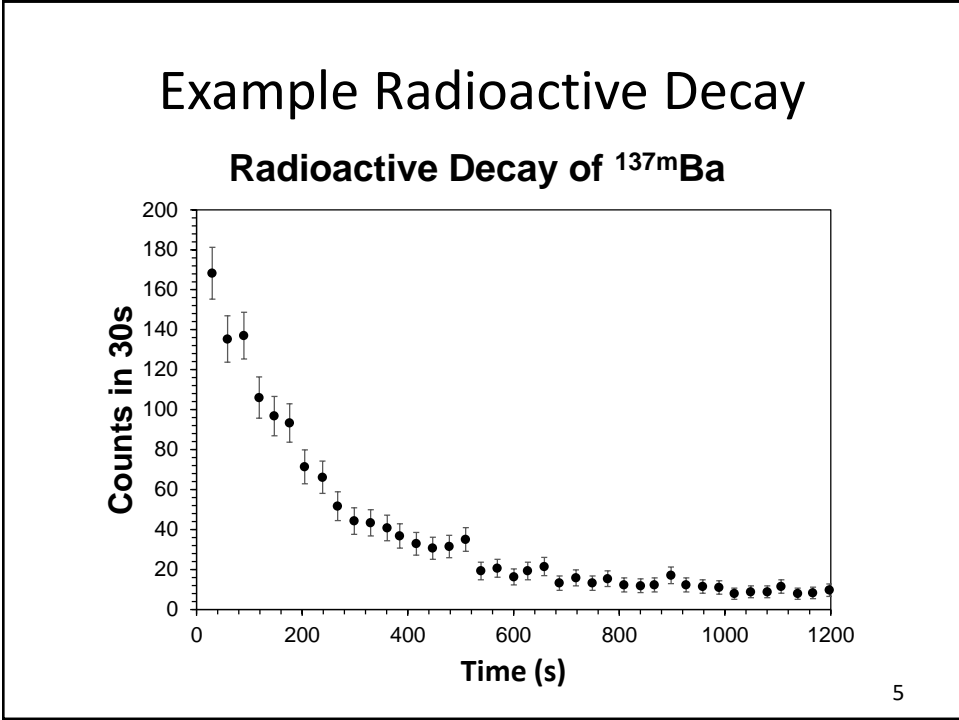
- Single site binding: $P + L \rightleftarrows PL$

$$\bar{v} = \frac{[PL]}{P_0} = \frac{(P_0 + L_0 + K) - \sqrt{(P_0 + L_0 + K)^2 - 4P_0L_0}}{2P_0}$$
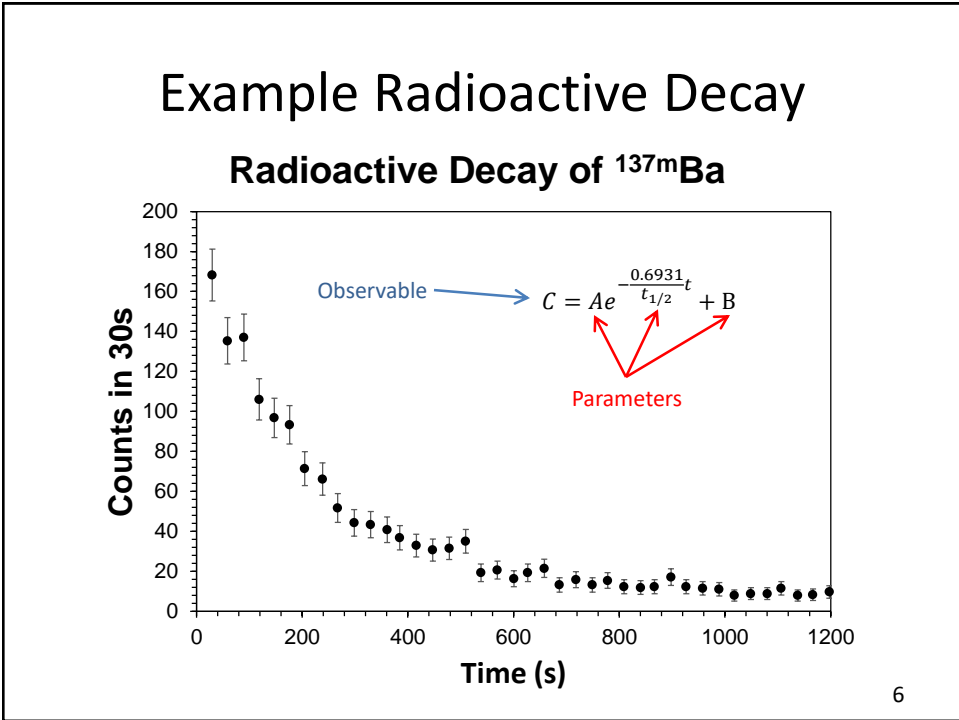
- Folding vs. Temperature: $N \rightleftarrows U$

$$f = \frac{1}{1 + K} = \frac{1}{1 + e^{-(\Delta\bar{H}^0 - T\Delta\bar{S}^0)/RT}}$$

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Example Radioactive Decay

Radioactive Decay of ^{137m}Ba

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Example Radioactive Decay

Radioactive Decay of ^{137m}Ba

Observable $\longrightarrow$ $C = Ae^{-\frac{0.6931}{t_{1/2}}t} + B$

Parameters

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How to Find Parameters?

- Minimize the difference between the observed and model-calculated values:
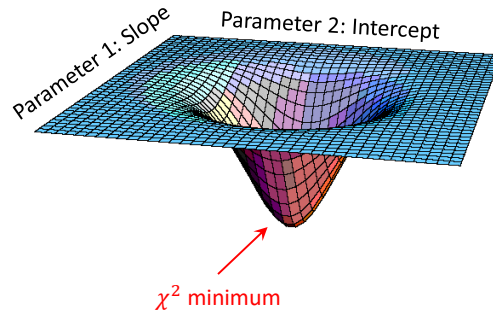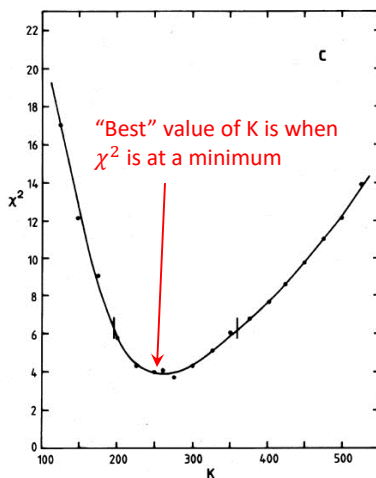
$$\chi_n^2 = \frac{1}{N_d} \sum_i \left\{ \frac{1}{\sigma_i^2} [y_i - f(x_i)]^2 \right\}$$

- Definitions:
  - y_i = observed data point i
  - $f(x_i)$ = model calculated point x_i (will change when parameters are changed)
  - σ_i = uncertainty for point i
  - N_d = # of observations - # of parameters

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Minimizing Chi-Square

"Best" value of K is when χ^2 is at a minimum

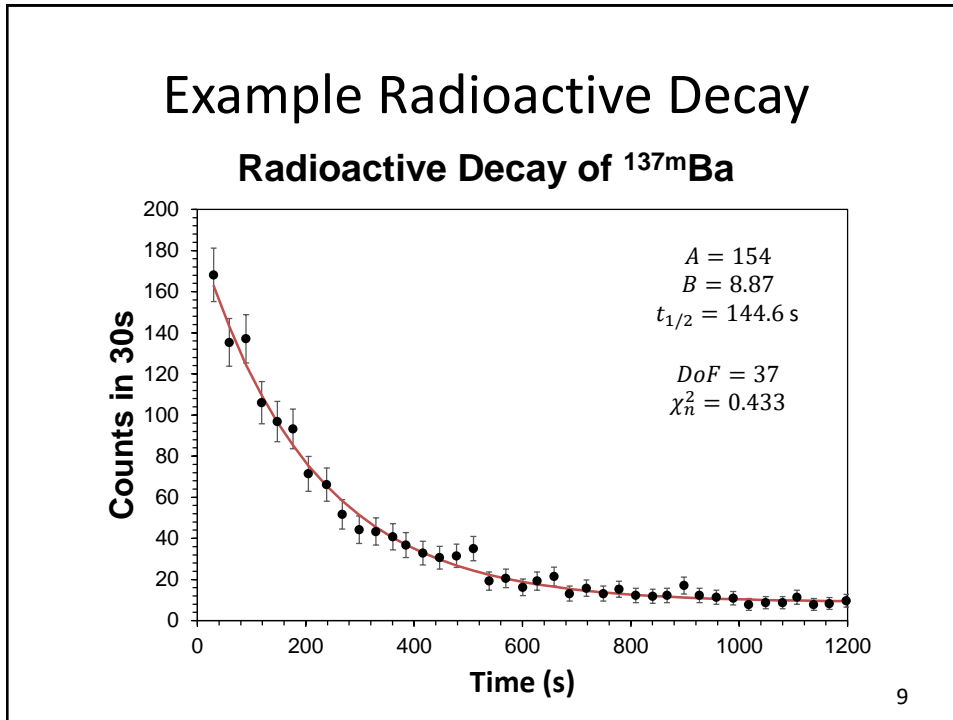Parameter 1: Slope   Parameter 2: Intercept

χ^2 minimum

- Minimizing χ^2 works for multiple parameters, too

- Finding the global minimum can be hard

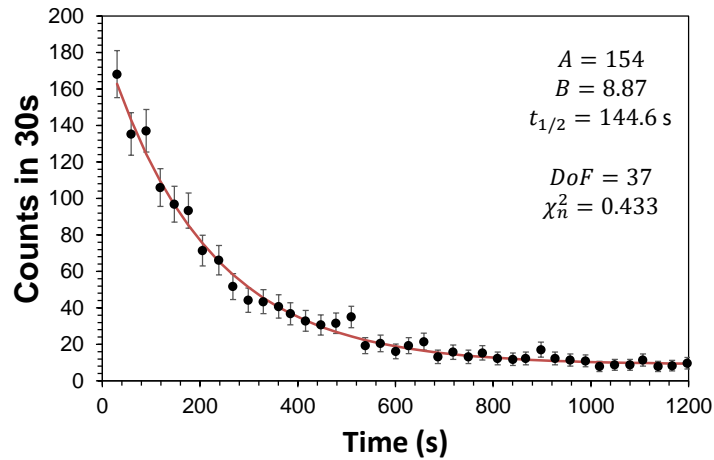https://ned.ipac.caltech.edu/level5/Wall2/Wal3_4.html

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Example Radioactive Decay

Radioactive Decay of ^{137m}Ba

$A = 154$
$B = 8.87$
$t_{1/2} = 144.6$ s

$DoF = 37$
$\chi_n^2 = 0.433$

Counts in 30s

Time (s)

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Think and Discuss

The accepted half-life for ^{137m}Ba is 153 s. Is our value of 144.6 good?

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How to Assess a Fit:
#1: Does the fit look good?

Radioactive Decay of ^{137m}Ba

$A = 154$
$B = 8.87$
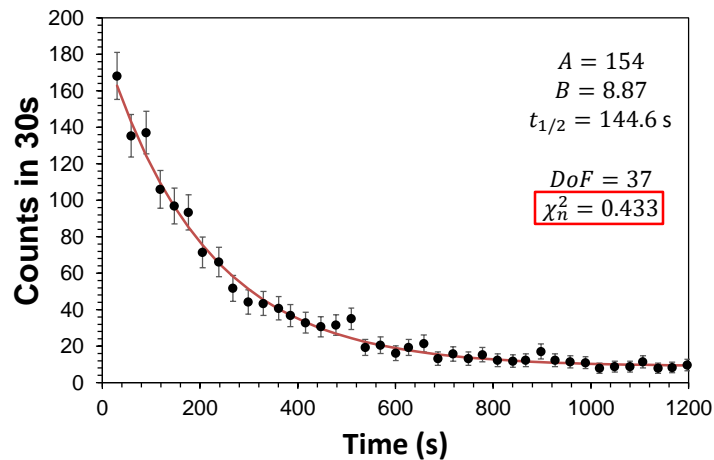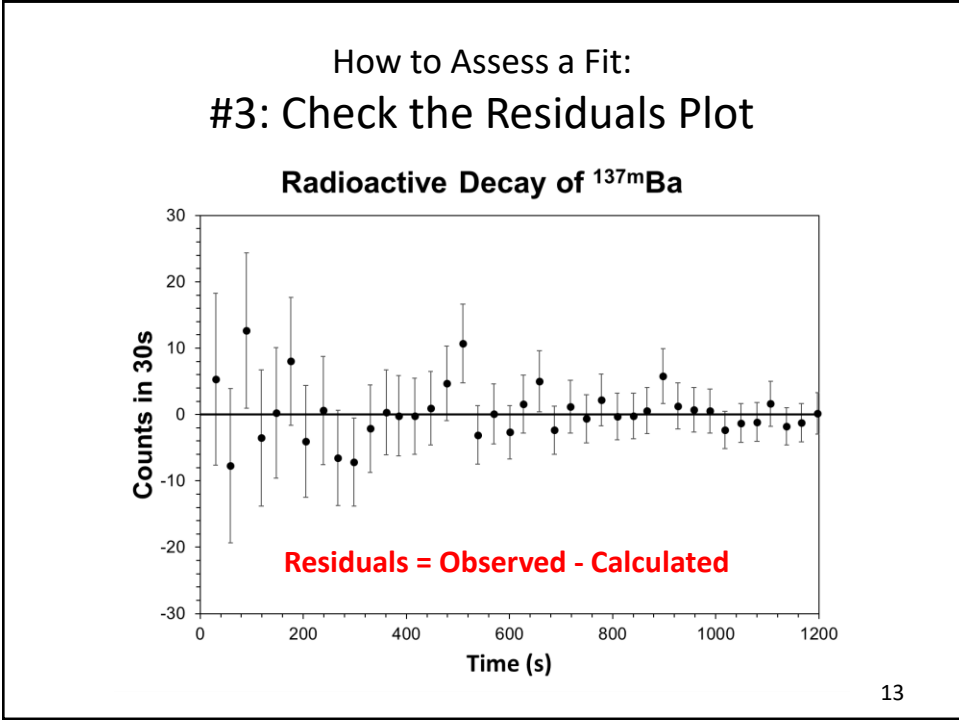$t_{1/2} = 144.6$ s

$DoF = 37$
$\chi_n^2 = 0.433$

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How to Assess a Fit:
#2: Is Normalized Chi-Square (χ_n^2) ≤ 1.0?

Radioactive Decay of ^{137m}Ba

$A = 154$
$B = 8.87$
$t_{1/2} = 144.6$ s

$DoF = 37$
$\chi_n^2 = 0.433$

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How to Assess a Fit:
#3: Check the Residuals Plot

Radioactive Decay of ^{137m}Ba

Residuals = Observed - Calculated

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Bad Fit Example:
Model Over-Predicts Data

Radioactive Decay of ^{137m}Ba

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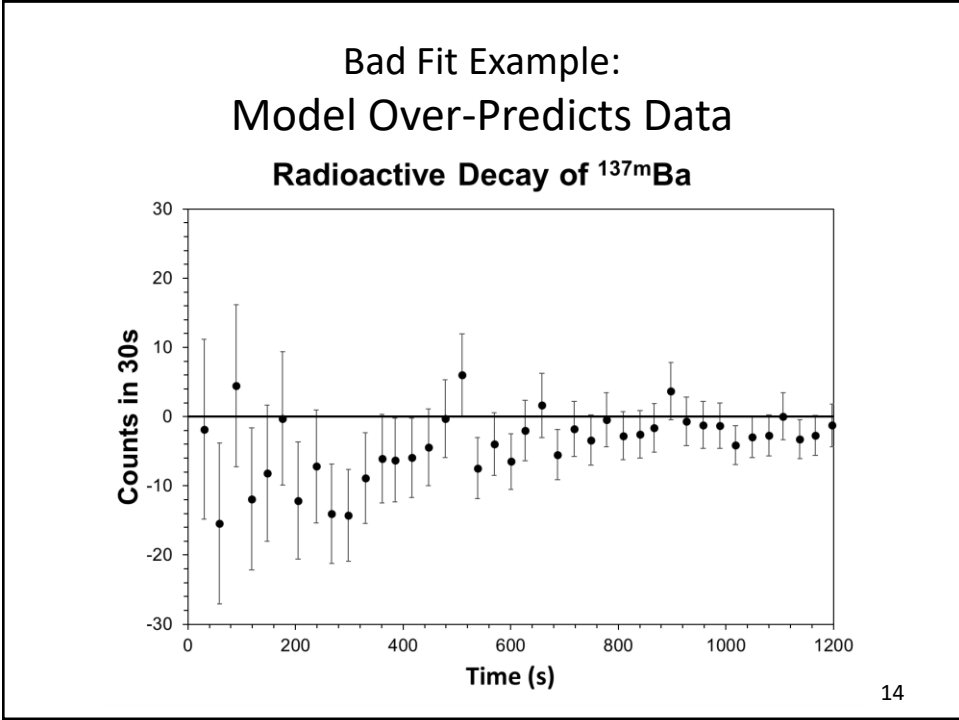# Bad Fit Example:
Systematic Trends in Residuals
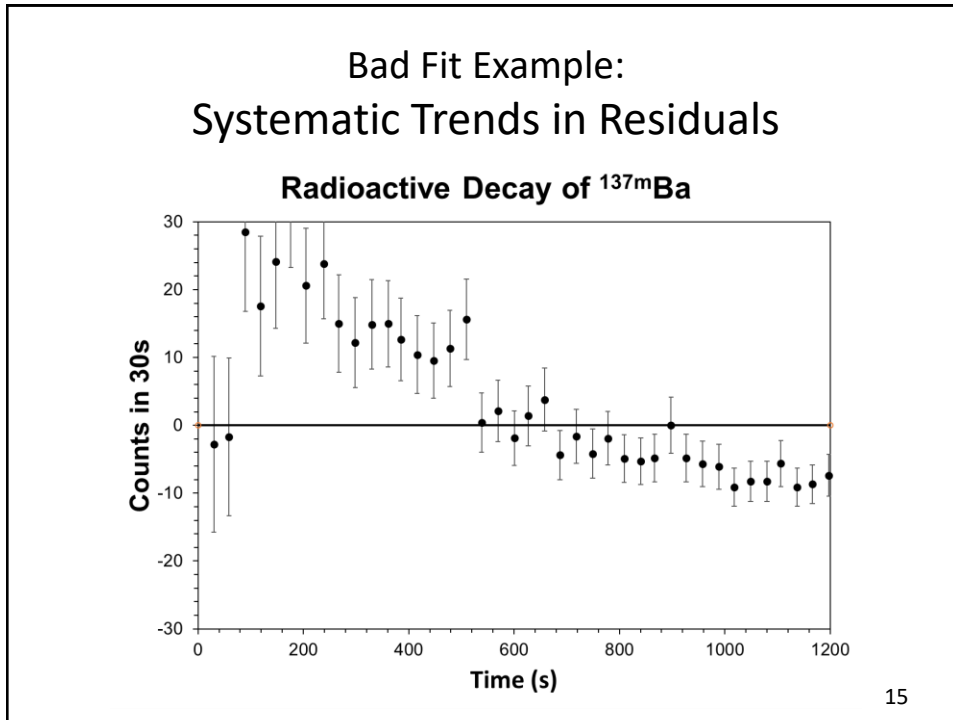
Radioactive Decay of ^{137m}Ba


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Biochemical Model Building

- **Step 1:** Come up with a hypothesis about how a system works
  - How many binding sites?
  - Is there cooperativity?

- **Step 2:** Translate the qualitative hypotheses into an observable mathematical form with *parameters*
  - Example parameters: K, tau, N
  - Parameters may not be known

- **Step 3:** Design an experiment that that can produce observables from step 2; perform the experiment
  - *Optimize* the parameters to make the fit look as good as possible

- **Step 4:** Assess the fit – Is the agreement convincing?
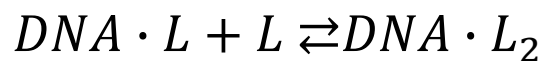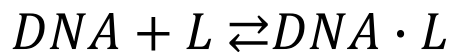
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Practical Example: DNA Binding

TMPyP4
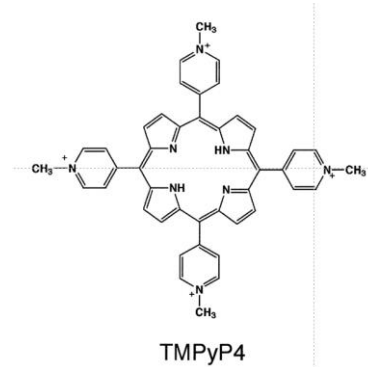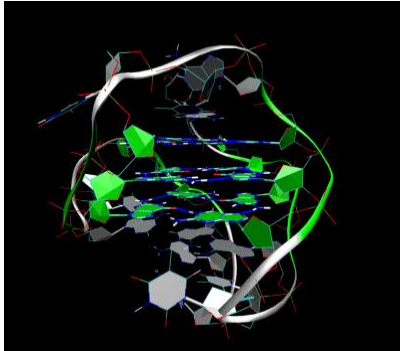
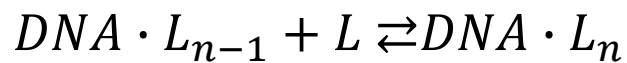- What is association constant?
- How many binding sites?

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Practical Example: DNA Binding

$$DNA + L \rightleftarrows DNA \cdot L$$
$$DNA \cdot L + L \rightleftarrows DNA \cdot L_2$$
$$...$$
$$DNA \cdot L_{n-1} + L \rightleftarrows DNA \cdot L_n$$

- Assumption in our model: All equilibrium constants are identical (equivalent binding)

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Practical Example: DNA Binding

- Experimentally, it is possible to measure the _degree of binding_ ($\bar{v}$) as a function of free drug concentration (L):

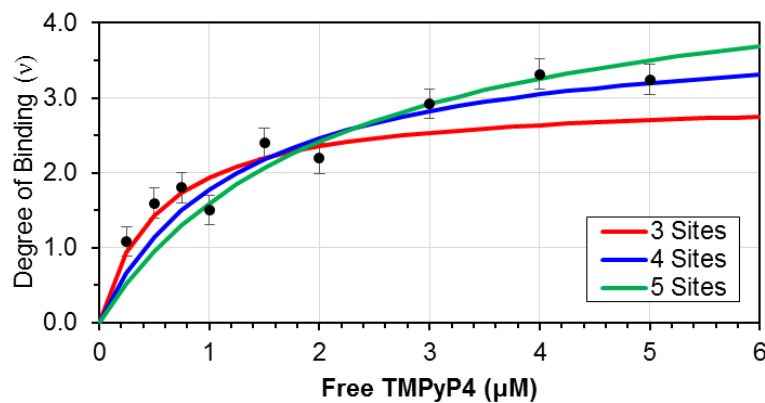$$\bar{v} = \frac{nK_{eq}L}{1 + K_{eq}L}$$

- We observe $\bar{v}$ and L; K_{eq} and n are the _parameters_.

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Practical Example: DNA Binding

Binding of TMPyP4 to Quadruplex DNA

- Download this data as `binding.txt` from the website.

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The Power of Gnuplot

- Minimizing chi-square can be very tricky; Excel isn't always up to the task

- What about estimating uncertainties of the parameters?

- Gnuplot is a tool that can handle some of these challenges (see tutorial)

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Summary

- Biochemical models allow us to quantify and predict the behavior of biological systems

- Fitting parameters allow us to optimize agreement between model and observations

- Programs like Gnuplot enable generalized model fitting with statistical analysis

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